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STOCHASTIC SEISMIC PERFORMANCE EVALUATION OF TUNED LIQUID COLUMN DAMPERS

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SUMMARY

The seismic performance of Tuned Liquid Column Dampers (TLCDs) for the passive control of flexible structures is investigated using random vibration analysis. A non-stationary stochastic process with frequency and amplitude modulation is used to represent the earthquake strong motion, and a simple equivalent linearization technique is used to account for the non-linear damping force in the TLCD. The governing equations of motion for the structure TLCD system are formulated and reduced to a first-order state vector equation, from which the differential equation for the system response covariance matrix is obtained. The TLCD performance is evaluated on the basis of selected structural response statistics, namely, the expected maximum and root-mean-square displacements, and root-mean-square absolute accelerations and interstorey shears. A parametric study and sensitivity analysis are conducted to assess the TLCD performance and identify critical design parameters. Illustrative examples are presented using SDOF and MDOF shear-beam structural models, a wide-banded stationary random base acceleration and two non-stationary random input ground motions representative of long- and short-duration ground accelerations with significant low-frequency content.

KEY WORDS: earthquake loading; passive control; random vibrations; structural dynamics; tuned dampers

INTRODUCTUION

Tuned Liquid Column Dampers (TLCDs) can be effective vibration control devices for flexible, tower-like structures subjected to long-duration, periodic or harmonic excitations,[1-3] and their potential application for earthquake engineering has recently been, explored.[4-6] TLCDs are a special type of Tuned Mass Dampers (TMDs) relying on the motion of a liquid mass in a tube-like container to counteract the external force while a Bernoullian-type orifice induces damping forces that dissipate energy. In this regard, TLCDs differ from Tuned Liquid Dampers (TLDs)[7-9] which rely upon the motion of a liquid in shallow rigid tanks to alter the dynamic properties of a structure and dissipate energy. As in TMDs and TLDs, the effectiveness of a TLCD depends on proper tuning and damping value; however, like TLDs and unlike traditional TMDs, the TLCD response is non-linear. In particular, the damping coefficient introduced by the orifice is response-dependent so that the optimum damping and tuning condition cannot be established *a priori* unless the loading level is known.

The description of TLCDs performance for generalized earthquake loadings is still lacking and their performance variability with respect to the design parameters is not established. In previous studies of TLCD performance under earthquake loadings, either time-history analysis using a few representative ground motions, or stationary random vibration analysis in the frequency domain were performed. The results obtained from these approaches are necessarily limited for a general description of system performance

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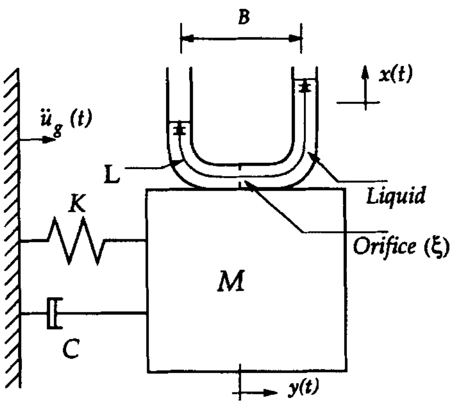
because they are strongly dependent on the particular ground motions considered when time-history analysis is used, and the assumption of stationarity is not, in general, a good representation of earthquake loadings.

A more realistic stochastic model of earthquake ground motions should be capable of representing their highly non-stationary nature. While several non-stationary stochastic ground motion models are available (e.g. References 10–12), the stochastic ground motion model with both frequency and amplitude modulation of Reference 13 is particularly well-suited for the random vibration analysis in the time domain. Here, time-domain random vibration analysis, together with a simple stochastic equivalent linearization technique is used to investigate the performance of TLCDs for the passive control of long-period structures, such as tall buildings and cable-stayed bridges,[14,15] under earthquake loadings. The study proposes a random vibration analysis and sensitivity analysis method well-suited for optimization studies, that are used to identify critical design parameters, and to investigate the validity of TLCDs as passive control devices as well as the sensitivity of their performance with respect to various design parameters.

TUNED LIQUID COLUMN DAMPER

A TLCD is a subsidiary vibration system consisting of a liquid mass in a tube-like container attached to the primary structure as shown in Figure 1. The restoring force resulting from the gravity forces acting on the



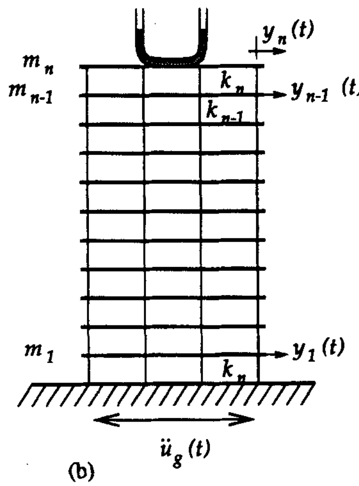


Figure 1. (a) SDOF system with TLCD and (b) shear-beam structure with TLCD

liquid mass, and the damping effects resulting from the hydrodynamic head loss when the liquid passes through an orifice smaller than the tube cross-section, reduce the vibration of the primary structure. In this respect, a TLCD may be regarded as an inertial vibration absorber.

Analytical model

Consider the simple structures and TLCD shown in Figure 1. The equation of motion for the liquid oscillation subjected to the absolute acceleration, $\ddot{y}_a$, of the container is given by[16]

$$\rho A L \ddot{x} + \frac{\rho A}{2} \xi |\dot{x}| \dot{x} + 2\rho A g x = -\rho A B \ddot{y}_a \tag{1}$$

where $\ddot{y}_a = \ddot{x}_g + \ddot{y}$, for the SDOF structure ($\ddot{y}_a = \ddot{x}_g + \ddot{y}_n$, for the MDOF strucuture), x is the vertical elevation change of the liquid surface, and ρ, L, B and A are the density, length, width and cross-sectional area of the liquid column, respectively. The constant ξ is the coefficient of head loss controlled by the opening ratio of the orifice[17] at the centre of the horizontal portion of the TLCD, and g is the acceleration of gravity. The undamped natural frequency of the TLCD is $\omega_T = \sqrt{2g/L}$, which depends only on the length of the oscillating liquid mass.

The equation of motion for the TLCD (equation (1)) can be combined with the equations of motion for the structure, which is modelled as an n-degrees-of-freedom (DOF) linear-elastic shear-beam-type structure. Accordingly, the equations of motion for the combined TLCD–structure system can be written as

$$\begin{bmatrix} \mathbf{m} & \mathbf{a} \\ \mathbf{a}^T & m \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{y}} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{0} \\ \mathbf{0} & c(t) \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{y}} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{0} \\ \mathbf{0} & k \end{bmatrix} \begin{Bmatrix} \mathbf{y} \\ x \end{Bmatrix} = - \begin{Bmatrix} \mathbf{mJ} \\ m_0 \end{Bmatrix} \ddot{u}_g \tag{2}$$

where $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ are the $(n \times n)$ mass, damping and stiffness matrices of the shear-beam structure except for element $\mathbf{m}_{nn}$ in the mass matrix, which is the sum of the nth storey mass and the TLCD mass; $\mathbf{y}$ is a n-dimensional vector of the storey displacements relative to the support; $\mathbf{J}$ is a n-dimensional vector of ones; $\mathbf{a} = [0 \ \cdots \ 0 \ m_0]^T$ where $m_0 = \rho AB$; $k = 2\rho Ag$; $\mathbf{0}$ is either a n-dimensional column or row vector of zeros as appropriate; $m = \rho AL$; and $c(t)$ is the velocity-dependent non-linear damping term for the TLCD given by $(\rho A/2)\xi|\dot{x}|$.

Using the normal mode approach[18] the displacement response vector for the structure can be approximated by

$$\mathbf{y} \approx \mathbf{\Phi}_r \mathbf{q} \tag{3}$$

where $\mathbf{\Phi}_r$ is an $(n \times m)$ truncated modal matrix containing those mode shapes of the structure that contribute most significantly to the response, usually the first m mode shapes and frequencies, and $\mathbf{q}$ is a m-dimensional vector of generalized modal displacements. Substituting equation (3) into equation (2) and premultiplying the result by

$$\begin{bmatrix} \mathbf{\Phi}_r^T & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}$$

the equations of motion (equation (2)) become

$$\begin{bmatrix} \mathbf{m}^* & \mathbf{\Phi}_r^T \mathbf{a} \\ \mathbf{a}^T \mathbf{\Phi}_r & m \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} \mathbf{c}^* & \mathbf{0} \\ \mathbf{0} & c(t) \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} \mathbf{k}^* & \mathbf{0} \\ \mathbf{0} & k \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ x \end{Bmatrix} = - \begin{Bmatrix} \mathbf{\Phi}_r^T \mathbf{mJ} \\ m_0 \end{Bmatrix} \ddot{u}_g \tag{4}$$

where $\mathbf{m}^*$, $\mathbf{c}^*$ and $\mathbf{k}^*$ are diagonal matrices, as a result of orthogonality. Classical damping is assumed for the structure with the added mass of the TLCD, but not for the combined structure–TLCD system. Any number of significant modes, not necessarily in order, may be considered in equation (4). In the following, two cases will be considered: in the first case, only the first mode is used which reduces the structure–TLCD system to a 2-DOF system, that can be used for some basic assessment of the TLCD performance under earthquake loadings, whereas, in the second case, the first m normal modes and frequencies will be considered for a more detailed assessment of the TLCD performance.

When only the first mode is considered, the mode shape $\mathbf{\Phi}_1$ is normalized such that $\Phi_{1n} = 1{\cdot}0$ (and $\mathbf{q}_1 \approx y_n$), thus obtaining

$$\begin{bmatrix} m_1^* & m_0 \\ m_0 & m \end{bmatrix} \begin{Bmatrix} \ddot{y}_n \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} c_1^* & 0 \\ 0 & c(t) \end{bmatrix} \begin{Bmatrix} \dot{y}_n \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} k_1^* & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} y_n \\ x \end{Bmatrix} = - \begin{Bmatrix} \mathbf{\Phi}_1^{\mathrm{T}} \mathbf{mJ} \\ m_0 \end{Bmatrix} \ddot{u}_g \tag{5}$$

which corresponds to the case of a SDOF structure and will be used for the initial assessment of the TLCD performance.

Stochastic equivalent linearization

The TLCD response is non-linear as a result of the drag-type forces induced by the orifice as shown with the second term of equation (1). For the random vibration analysis the equation of motion for the TLCD (equation (1)) is linearized as follows. First, the following equivalent auxiliary linear system is chosen to represent the original non-linear system:[19]

$$\rho AL\ddot{x}(t) + c_{\mathrm{eq}}\dot{x}(t) + 2\rho Agx(t) = -\rho AB\ddot{y}(t) \tag{6}$$

where c_{eq} is the equivalent linear damping coefficient for the TLCD. By minimizing the mean-square error between equations (1) and (6), and assuming that $x(t)$ is a Gaussian process[6,20,21]

$$c_{\mathrm{eq}} = \sqrt{\frac{2}{\pi}}\, \rho A \xi \sigma_{\dot{x}} \tag{7}$$

where $\sigma_{\dot{x}}$ is the standard deviation of the liquid mass velocity $\dot{x}(t)$. Note that c_{eq} depends on $\sigma_{\dot{x}}$, which is not known *a priori*.

STOCHASTIC RESPONSE ANALYSIS

For the stochastic response analysis the equations of motion for the structure–TLCD system are augmented with the filter equations for the ground motion and then written in a state-vector form, from which the response covariance matrix can be obtained.[22-25]

Ground motion model

The stochastic and transient characteristics of the earthquake input ground accelerations are modelled by a non-stationary stochastic process with both frequency and amplitude modulation.[13] The filtering equations for the ground motion are

$$\ddot{x}_g + \left[-\frac{\phi''}{\phi'} + 2\zeta_g \omega_g \phi' \right] \dot{x}_g + [\omega_g \phi']^2 x_g = -I(t)[\phi']^2 \zeta[\phi(t)] \tag{8}$$

$$\ddot{x}_f + \left[-\frac{\phi''}{\phi'} + 2\zeta_f \omega_f \phi' \right] \dot{x}_f + [\omega_f \phi']^2 x_f = 2\zeta_g \omega_g \phi' \dot{x}_g + [\omega_g \phi']^2 x_g \tag{9}$$

where $\zeta(\phi)$ is a Gaussian white noise in the ϕ-scale, ω_g, ζ_g, ω_f and ζ_f are filter parameters identified below, $\phi(t)$ is the frequency modulation function (a smooth function of time, t); and $\phi'(t)$ and $\phi''(t)$ are the first and second derivatives of $\phi(t)$, respectively. The input ground acceleration to the structure is

$$2\zeta_g \omega_g \frac{\dot{x}_g}{\phi'} + \omega_g^2 x_g - 2\zeta_f \omega_f \frac{\dot{x}_f}{\phi'} - \omega_f^2 x_f \tag{10}$$

with Power Spectral Density (PSD)

$$S(\omega, t) = \frac{1}{\phi'(t)} S_{\mathrm{CP}}\left(\frac{\omega}{\phi'} \right) \tag{11}$$

where $S_{CP}(\omega)$ is the Clough–Penzien[26] PSD given by

$$S(\omega) = S_0\left[\frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2}\right]\left[\frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2}\right] \tag{12}$$

in which S_0 is the spectral intensity of the white noise. The amplitude modulation function, $I(t)$, is a deterministic ground motion intensity envelope function given by

$$I^2(t) = a\frac{t^b}{d + t^e}\exp(-ct) \tag{13}$$

where a, b, c, d and e are coefficients to be determined.

If needed, the ground motion can be modelled by the sum of two stochastic processes such as the one represented above where, for example, one stochastic process represents the low-frequency component of the ground motion whereas the other represents the high frequency component of the ground motion.[25]

State equation and response statistics

The equations of motion (equation (4)) can be written in a more compact form as follows:

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = -\mathbf{M}'\ddot{u}_g \tag{14}$$

where $\mathbf{Y} = [\mathbf{q}\, x]^T$, which can then be reduced to the first-order form

$$\begin{Bmatrix}\dot{\mathbf{Y}}\\ \ddot{\mathbf{Y}}\end{Bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\begin{Bmatrix}\mathbf{Y}\\ \dot{\mathbf{Y}}\end{Bmatrix} - \begin{Bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{M}'\end{Bmatrix}\ddot{u}_g \tag{15}$$

The filtering equations for the ground motion can be added to equation (15), to obtain

$$\dot{\mathbf{Z}} = \mathbf{G}\mathbf{Z} + \mathbf{b} \tag{16}$$

where

$$\mathbf{Z} = [x_g \dot{x}_g x_f \dot{x}_f \mathbf{Y}\dot{\mathbf{Y}}]^T \tag{17}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 & 0 & \mathbf{0} & \mathbf{0}\\ -(\omega_g\phi')^2 & \dfrac{\phi''}{\phi'} - 2\zeta_g\omega_g\phi' & 0 & 0 & \mathbf{0} & \mathbf{0}\\ 0 & 0 & 0 & 1 & \mathbf{0} & \mathbf{0}\\ (\omega_g\phi')^2 & 2\zeta_g\omega_g\phi' & -(\omega_f\phi')^2 & -\dfrac{\phi''}{\phi'} - 2\zeta_g\omega_g\phi' & \mathbf{0} & \mathbf{0}\\ 0 & 0 & 0 & 0 & \mathbf{0} & \mathbf{I}\\ -\tilde{\mathbf{M}}\omega_g^2 & -\tilde{\mathbf{M}}\dfrac{2\zeta_g\omega_g}{\phi'} & \tilde{\mathbf{M}}\omega_f^2 & \tilde{\mathbf{M}}\dfrac{2\zeta_f\omega_f}{\phi'} & -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{18}$$

in which $\tilde{\mathbf{M}} = -\mathbf{M}^{-1}\mathbf{M}'$, $\mathbf{0}$ is an $(m + 1)$ row or column vector as appropriate, and $\mathbf{b}$ is a $(2m + 6)$ vector, whose only non-zero element is $\mathbf{b}_2 = -\zeta(t)I(t)\phi'^2$. The number of unknowns is $(2m + 6)$.

Postmultiplying equation (16) by $\mathbf{Z}^T$, adding the result to its transpose, and taking expected values, yields the following *Liapunov* matrix differential equation:

$$\dot{\mathbf{S}} = \mathbf{G}\mathbf{S} + \mathbf{S}\mathbf{G}^T + \mathbf{B} \tag{19}$$

where

$$\mathbf{S} = E[\mathbf{Z}\ \mathbf{Z}^T] \tag{20}$$

is the covariance matrix, $\dot{\mathbf{S}}$ is the time-derivative of $\mathbf{S}$, and all $\mathbf{B}_{ij} = 0$, except $\mathbf{B}_{22}$ which is given by

$$\mathbf{B}_{22} = 2\pi S_0 I^2(t)[\phi'(t)]^3 \tag{21}$$

The covariance matrix $\mathbf{S}$, contains all the mean-square response statistics of interest for this study. Note that the matrix differential equation for the covariance matrix is non-linear because the constant c_{eq} for the TLCD depends on $\sigma_{\dot{x}}^2$, which is an element of the response covariance matrix. The differential equation for the response covariance matrix is solved by standard procedures (e.g. the Adams–Moulton method[27]).

The response statistics used in this study are: the expected maximum values of the structure and TLCD displacements, and the root-mean-square (RMS) of the displacements, absolute accelerations and interstorey shears; and the time-average of those response statistics.

The expected maximum values of the structure and TLCD displacements are computed as the threshold values, y_{max} or x_{max}, for which the integral of the crossing rates $\nu(y_{max}, t)$ and $\nu(x_{max}, t)$, over the entire loading duration are equal to 1·0,[25] i.e.

$$\int_0^T \nu(y_{max}, t)\, dt = 1{\cdot}0, \qquad \int_0^T \nu(x_{max}, t)\, dt = 1{\cdot}0$$

The RMS values of interstorey drift can conveniently be obtained from the covariance matrix $\mathbf{S}$. Let Δ_i be the ith storey drift (i.e. $\Delta_i = \mathbf{y}_i - \mathbf{y}_{i-1}$) which can be expanded using equation (3). Then, the mean-square of Δ_i is expressed as

$$E[\Delta_i^2] = \sum_{j=1}^{m} \sum_{k=1}^{m} (\phi_{i,j} - \phi_{i-1,j})(\phi_{i,k} - \phi_{i-1,k}) E[\mathbf{q}_j \mathbf{q}_k], \quad i = 2, \ldots, n \tag{22}$$

$$E[\Delta_1^2] = \sum_{j=1}^{m} \sum_{k=1}^{m} \phi_{1,j} \phi_{1,k} E[\mathbf{q}_j \mathbf{q}_k] \tag{23}$$

where $\phi_{i,j}$ is the jth mode shape corresponding to ith storey; $\mathbf{q}_i$ is the ith modal displacement; and m is the number of reduced modes. Note that $E[\mathbf{q}_j \mathbf{q}_k]$ can be obtained directly from the covariance matrix.

Similarly, the absolute acceleration of the structure ($\ddot{\mathbf{y}}_{a,i} = \ddot{\mathbf{y}}_i + \ddot{u}_g$) can be obtained using equation (3) and the relationship $\ddot{\mathbf{q}}_j = (\mathbf{gz})$ reflected in the bottom m equations of equation (15), where $\mathbf{g}$ and $\mathbf{z}$ are the appropriate submatrix of $\mathbf{G}$ and subvector of $\mathbf{Z}$, respectively. Then, it can be shown that

$$\begin{aligned} E[\ddot{\mathbf{y}}_{a,i}^2] = {} & \sum_{j=1}^{m} \sum_{l=1}^{m} \phi_{i,j} \phi_{i,l} \sum_{k=1}^{n} \sum_{p=1}^{n} \mathbf{g}_{j,k} \mathbf{g}_{l,p} E[\mathbf{z}_k \mathbf{z}_p] - 2 \sum_{j=1}^{m} \phi_{i,j} \sum_{k=1}^{n} \sum_{l=1}^{4} \mathbf{g}_{j,k} \frac{\mathbf{g}_{*,l}}{\alpha} E[\mathbf{z}_k \mathbf{z}_l] \\ & + \sum_{j=1}^{4} \sum_{k=1}^{4} \frac{\mathbf{g}_{*,j}}{\alpha} \frac{\mathbf{g}_{*,k}}{\alpha} E[\mathbf{z}_j \mathbf{z}_k] \end{aligned} \tag{24}$$

where the subscript $*$ denotes any row corresponding to the bottom m equations of equation (16); n in equation (24) is the order of equation (16) which is $2m + 6$; α is the value of $\tilde{\mathbf{M}}$ corresponding to $*$; and $E[\mathbf{z}_j \mathbf{z}_k]$ are once again obtained from the $\mathbf{S}$ matrix.

ILLUSTRATIVE EXAMPLES

The TLCD performance as a passive control system for flexible structures under random earthquake loadings is illustrated using SDOF and MDOF shear-beam structural models and three input random ground accelerations. First, wide-banded and stationary input random ground accelerations are considered to investigate the basic TLCD performance, to identify critical and optimal design parameters and to assess the sensitivity of its performance with respect to the critical design parameters. Because most earthquake ground motions are highly transient and TLCDs are expected to be effective for long-period structures, their performance is evaluated using simple structural models and two non-stationary random input ground motions representative of long- and short-duration ground accelerations with significant low-frequency content.

Sensitivity analysis

Critical design parameters for a TMD are, generally, the mass ratio, $\mu = m/M$, the tuning ratio, $\gamma = \omega_d/\omega_s$ and the damping ratio ζ_d. For lightly damped structures, when their damping is neglected, optimal tuning ratios and damping ratios expressed as a function of the mass ratio have been proposed by Den Hartog[28] and Warburton[29] for steady-state sinusoidal loads and for stationary white-noise loads, respectively. Under those conditions, the optimal tuning ratio slowly decreases from about 1·0 to 0·9, when the mass ratio increases from about 0 to 0·10, whereas the optimal damping ratio increases from approximately 0·025 to 0·15 when the mass ratio increases from about 0 to 0·10. For ratios up to 0·04 it appears that the damping ratio is the more critical parameter provided that the tuning ratio is between 1·0 and 0·95. Since the TLCDs are non-linear, their optimal design parameters will depend on the loading intensity. Moreover, of direct interest for the TLCD design is the coefficient of head loss, ξ, which can be related to the damping of the equivalent linear system, c_{eq}, given by equation (7). When the structural damping is not ignored, the selection of the optimal design parameters, γ_{opt} and ξ_{opt}, becomes a two-dimensional optimization problem that may need to be solved for each given loading.

For stationary white-noise input base accelerations the response covariance matrix is a symmetric 4×4 matrix and the equation for the response covariance matrix is reduced to a simple algebraic equation that can be solved easily. Moreover, the derivative (sensitivity) of the mean-square covariance matrix with respect to a given design parameter, p, can be obtained easily by differentiating equation (19) as follows:[24]

$$\frac{\partial}{\partial t}[\mathbf{S}_{,p}] = \mathbf{G}_{,p}\mathbf{S} + \mathbf{S}\mathbf{G}^{\mathrm{T}}_{,p} + \mathbf{G}\mathbf{S}_{,p} + \mathbf{S}_{,p}\mathbf{G}^{\mathrm{T}} + \mathbf{B}_{,p} \tag{25}$$

where $\mathbf{G}_{,p}$ contains the derivatives of the elements of $\mathbf{G}$ with respect to p, which can be obtained in closed form. For the stationary case, the derivative equation is further simplified to an algebraic equation of the form

$$\mathbf{G}\mathbf{S}_{,p} + \mathbf{S}_{,p}\mathbf{G}^{\mathrm{T}} = -\mathbf{B}_{,p} - \mathbf{G}_{,p}\mathbf{S} - \mathbf{S}\mathbf{G}^{\mathrm{T}}_{,p} \tag{26}$$

The derivative equations given above are linear and can easily be solved once the response covariance matrix, S, is obtained. Having a set of equations for the response covariance matrix and its derivatives with respect to the design parameters of interest allows for the use of appropriate optimization algorithms to find the TLCD design parameters that will minimize the structural response of interest while satisfying required design constraints. For an SDOF structure under a stationary white-noise base excitation, the solution for S can be obtained easily so that a simple parametric study generally suffices to find the optimal design parameters. As an example, the optimal coefficient of head loss, ξ_{opt}, for an SDOF structure with $D = 0{\cdot}01$ and $D = 0{\cdot}02$, and two white-noise intensities are shown in Table I for $B/L = 0{\cdot}7$ and $\gamma = 1{\cdot}0$. The results in Table I indicate that optimal values of the coefficient of head loss depend on the mass ratio, μ, on the structural damping ratio, D, and on the loading intensity, S_0. However, it is interesting to notice that the equivalent viscous damping ratios for the linearized governing equation of motion for the TLCD, i.e. equation (6), corresponding to the optimal coefficients of head loss shown in Table I, primarily depend on the mass ratio, μ, and are approximately equal to 0·05 or 0·07 for the mass ratios of 0·02 and 0·04, respectively.

Table I. Optimal coefficient of head loss for white-noise loading

		$D = 0{\cdot}01$	$D = 0{\cdot}02$
S_0 (cm^2/cm^3)	μ	ξ_{opt}	ξ_{opt}
32·3	0·02	0·92	1·05
	0·04	2·10	2·30
16·15	0·02	1·30	1·50
	0·04	2·90	3·30

To illustrate further the TLCD performance, the structural displacement response reduction ratio, i.e. the ratio of the expected maximum structural responses with and without a TLCD, is computed for an SDOF structure with a viscous damping ratio, $D = 0{\cdot}02$, and a natural frequency, $\omega_s = 2{\cdot}0$ rad/sec, under a wide-banded stationary filtered Gaussian white noise. The power spectral density of the input ground acceleration is the Clough–Penzien PSD (see equation (12)) with $\omega_g = 10{\cdot}9$ rad/sec, $\zeta_g = 0{\cdot}96$, $\omega_f = 1{\cdot}5$ rad/sec and $\zeta_f = 0{\cdot}9$. These values of ω_g and ζ_g have been proposed in Reference 24 for soft soil sites. The results obtained are shown in Figure 2(a) for mass ratios of 0·02 and 0·04, $\gamma = 1{\cdot}0$ and $B/L = 0{\cdot}7$, and for two different load intensities. For this example, the structural response reduction is essentially a result of the tuned mass damper effect, because of negligible structural response change when the mass of the TLCD is added to the mass of the structure.

The maximum displacements, $x_{\max}$, of the liquid column are shown in Figure 2(b). It is important to notice that if the TCLD response becomes excessively large, the geometry of the oscillating liquid mass will change and the equations of motion used are no longer valid. The limiting value of the maximum TLCD response depends on the length $L = 2g/\omega_d^2$ and on the ratio B/L. The length L controls the frequency of the TLCD and, therefore, the tuning ratio. For the example analysed, $L = 491$ cm and $\gamma = 1{\cdot}0$. For this length of the oscillating liquid mass and $B/L = 0{\cdot}7$, the maximum TLCD displacement cannot exceed $(L - B)/2 = 73{\cdot}6$ cm. This condition ensures that the horizontal portion of the TLCD is always filled with liquid. If this condition is violated, the mass coupling in equation (5) reduces, and the effectiveness of the TLCD decreases. Since the length of the liquid column is inversely proportional to the square of the damper frequency, this condition becomes less important as the periods of the modes of vibration to be controlled increase. Mechanisms and procedures to reduce the limiting effect of excessive TLCD displacements and the subsequent loss of inertial coupling are currently under investigation.

Sensitivity coefficients of the root-mean-square structural displacement response, σ_y, with respect to two design parameters and for the wide-banded loading under consideration are shown in Figures 3(a) and 3(b). Those sensitivity coefficients, which are computed using equations (25) and (26), are: (a) partial derivatives of σ_y with respect to the natural logarithm of the coefficient of head loss, $\sigma_{y,\ln\xi}$, for $\gamma = 1{\cdot}0$; and (b) partial derivatives of σ_y with respect to the tuning ratio, $\sigma_{y,\gamma}$, with ξ equal to the optimal values obtained in part (a). For both cases $B/L = 0{\cdot}7$. The derivative with respect to ln ξ, which is $\sigma_{y,\ln\xi} = \xi\sigma_{y,\xi}$, is shown because its plot permits a more clear identification of the optimal value of ξ than the plot of the derivative with respect to ξ. Also shown in Figure 3 are the values of σ_y for the optimal values of ξ and γ in Figures 3(a) and 3(b), respectively.

Optimal tuning ratios appear to be close but slightly less than 1·0. The results shown in Figures 2 and 3 and in Table I indicate that the optimal values of ξ increase when the mass ratios increase and when the load intensity decreases, and that optimal tuning ratios for a fixed value of ξ and B/L are slightly less than unity and decrease when the mass ratio increases.

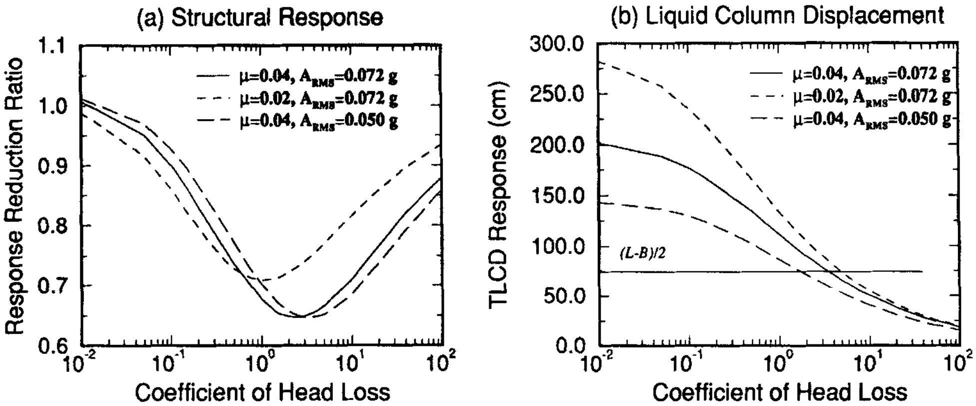


Figure 2. Stationary wide-banded load: (a) response reduction ratio; (b) expected maximum TLCD displacements

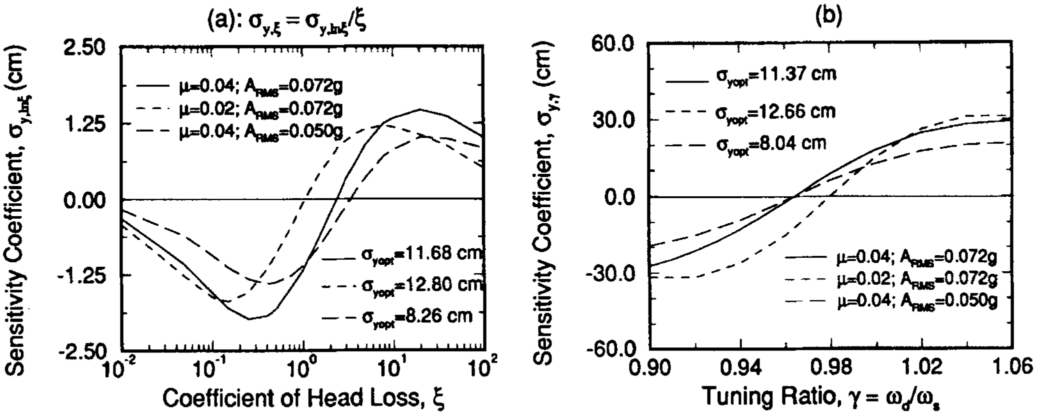


Figure 3. Sensitivity coefficients: (a) sensitivity with respect to the coefficient of head loss; (b) sensitivity with respect to the tuning ratio

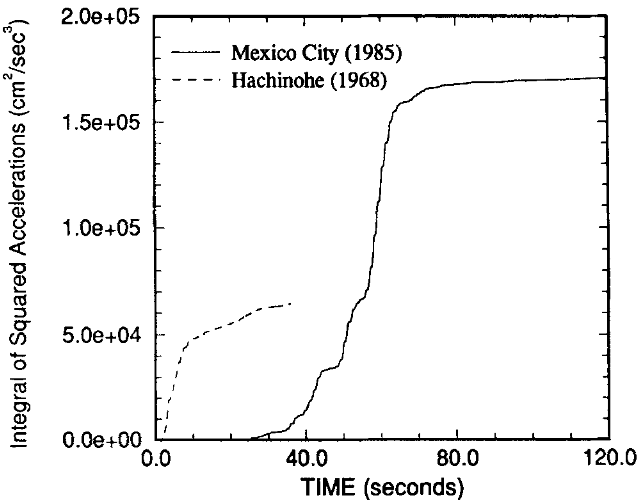


Figure 4. Time integral of squared accelerations for 1895 Mexico City earthquake (S60E-SCT) and E–W component of 1968 Hachinohe earthquake (serial number 680229)

Non-Stationary input ground motions

The parameters for the stochastic ground motion model are selected such that its mean response spectrum for 1 per cent damping ($D = 0{\cdot}01$) approximates the response spectrum for the ground motion record (S60E-SCT) of the 1985 Mexico City earthquake and the E–W component of the 1968 Hachinohe record (serial number 680229). The 1985 Mexico City record is representative of long-duration ground motions with low-frequency content whereas the 1968 Hachinohe record is representative of short duration ground motions with low-frequency content. The integrals of the squared accelerations for both records are shown in Figure 4. The results shown in this figure clearly show the differences expected for the intensity modulation function of both ground motions, $I(t)$. It is also noted that the expected maximum ground accelerations for the 1968 Hachinohe record occur between 3·0 and 9·0 sec.

The values for the parameters for the stochastic ground motion models obtained with the criterion stated above are shown in Table II (Mexico City record) and in Table III (Hachinohe record). The parameters for the stochastic ground motion model obtained from the Mexico City record are similar but not equal to those reported by Yeh and Wen.[30] The stochastic ground motion model obtained from the 1968 Hachinohe record

Table II. Ground motion parameters (1985 Mexico City record)

ω_g (rad/sec)	ζ_g	ω_f (rad/sec)	ζ_f	S_0 (cm^2/sec^3)
4·1	0·15	3·1	0·035	$3{\cdot}92 \times 10^{-4}$

Frequency modulation function, $\phi(t)$

$$\phi(t) = 1{\cdot}61 - 9{\cdot}57 \times 10^{-3} t + 3{\cdot}29 \times 10^{-5} t^2$$

Intensity modulation function, $I(t)$

$$I^2(t) = 10{\cdot}97 \times 10^{40} \frac{t^{-0{\cdot}894}}{7{\cdot}14 \times 10^{38} + t^{21{\cdot}91}} \exp(0{\cdot}154t)$$

Table III. Ground motion parameters (1968 Hachinohe earthquake)
(a) Low frequency

ω_g (rad/sec)	ζ_g	ω_f (rad/sec)	ζ_f	S_0 (cm^2/sec^3)
2·39	0·04	0·87	0·30	$7{\cdot}93 \times 10^{-3}$

Frequency modulation function, $\phi(t)$

$$\phi(t) = 0{\cdot}983 + 4{\cdot}10 \times 10^{-3} t - 1{\cdot}19 \times 10^{-4} t^2$$

Intensity modulation function, $I(t)$

$$I^2(t) = 20{\cdot}84 \times 10^{4} \frac{t^{17{\cdot}596}}{1{\cdot}434 \times 10^{11} + t^{19{\cdot}569}} \exp(0{\cdot}000365t)$$

(b) High frequency

ω_g (rad/sec)	ζ_g	ω_f (rad/sec)	ζ_f	S_0 (cm^2/sec^3)
15·4	0·40	4·80	0·095	$4{\cdot}97 \times 10^{-3}$

Frequency modulation function, $\phi(t)$

$$\phi(t) = 0{\cdot}945 + 1{\cdot}242 \times 10^{-2} t - 6{\cdot}852 \times 10^{-5} t^2$$

Intensity modulation function, $I(t)$

$$I^2(t) = 16{\cdot}03 \times 10^{5} \frac{t^{17{\cdot}920}}{4{\cdot}509 \times 10^{8} + t^{20{\cdot}561}} \exp(0{\cdot}0881t)$$

is the sum of two stochastic processes, one representing the low frequency of the ground accelerations and the other representing the high-frequency content of the ground accelerations. The mean response spectra, for 1 per cent damping, $D = 0{\cdot}01$, for 10 simulated ground motions, and the corresponding response spectrum for the 1985 Mexico City record are shown in Figure 5, whereas the expected maximum dispacement response spectrum for $D = 0{\cdot}01$ from the stochastic model and from the 1968 Hachinohe record are shown in Figure 6.

Both SDOF and MDOF shear-beam type structural models are considered. The MDOF shear-beam type structural model is representative of a 30-storey building. The relevant modal parameters for the first five modes are shown in Table IV. A modal damping ratio of 2 per cent is assumed for all modes.

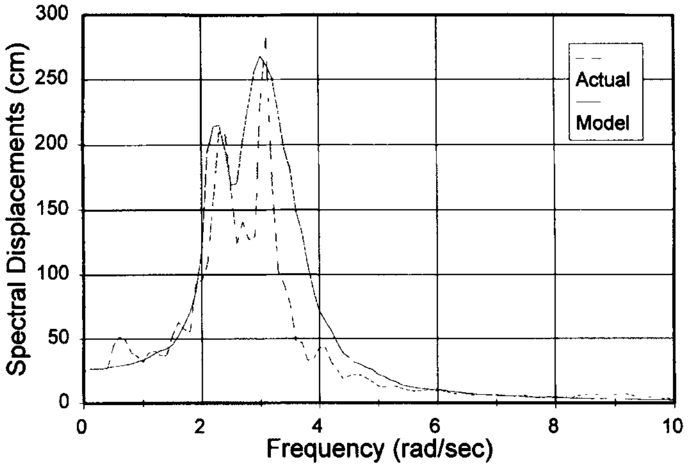


Figure 5. Spectral displacements (1 per cent damping ratio) of 1985 Mexico City earthquake (S60E-SCT) and simulated ground motions from fitted stochastic model

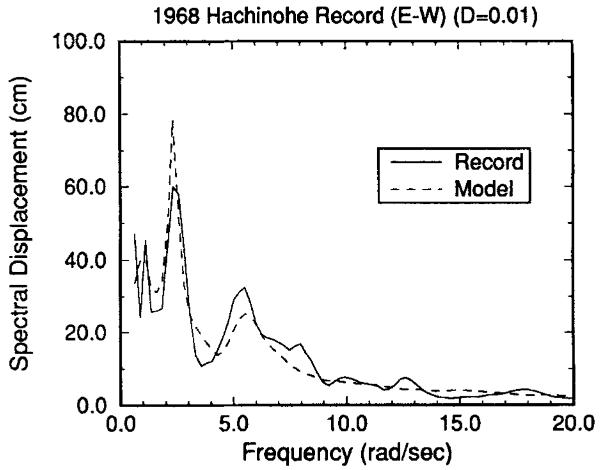


Figure 6. Spectral displacements (1 per cent damping ratio) of 1968 Hachinohe record (serial number 680229) and expected maximum displacements from fitted stochastic model

Table IV. Modal parameters for MDOF shear-beam structure

Mode number i	ω_i (rad/sec)	M_i^* (10^3 kg)	K_i^* (kN/m)	Φ_i^TmJ (10^3 kg)
1	2·20	725	3,521	946
2	6·30	660	26,180	314
3	10·4	687	74,403	193
4	14·5	748	158,100	144
5	18·6	713	247,260	109

Long-duration loading

The TLCD performance is evaluated first for a simple 2-DOF system where the first degree of freedom is thought of as representing the first mode of vibration of the structure and the second degree of freedom represents the motion of the TLCD. As before the TLCD performance is measured by the expected

maximum structural displacement reduction ratio, i.e. the ratio of the expected maximum structural displacement computed with and without the TLCD. The natural frequency of the structure is 2·2 rad/sec, which approximately corresponds to the first peak of the response spectra shown in Figure 5, and the TLCD properties except for those otherwise noted are $\gamma = 1{\cdot}0$ and $B/L = 0{\cdot}7$.

Expected maximum structural displacement reduction ratios are shown in Figure 7(a) as a function of the coefficient of head loss, ξ, for a mass ratio $\mu = 0{\cdot}04$, and for three different values of the loading intensity. The load factor, LF, shown in Figure 7 multiplies the intensity of the input power spectral densities and is proportional to the mean-square input ground acceleration. For this example, a very small response reduction (about 5 per cent) can be attributed to structural frequency change resulting from the added mass of the damper. As previously noted for the stationary input base motion, the optimum coefficient of head loss increases as the loading intensity decreases. The expected maximum displacements of the liquid column are shown in Figure 7(b). Since the motion of the liquid column is restricted to $(L - B)/2 = 60{\cdot}8$ cm, optimal TLCD performance cannot be achieved for this example. Nevertheless, a significant reduction of the expected maximum structural displacement response can still be obtained for values of ξ greater than the optimum ones. In passing, it is noted that since the tuning ratio is kept equal to 1·0, the TLCD performance shown in Figure 7(a) is not optimal with respect to both γ and μ.

The effect of the mass ratio on the TLCD performance is illustrated with the results shown in Figures 8(a) and 8(b). Shown in Figure 8(a) are the response reduction ratios for mass ratios of 0·02 and 0·04, 1 and 2 per

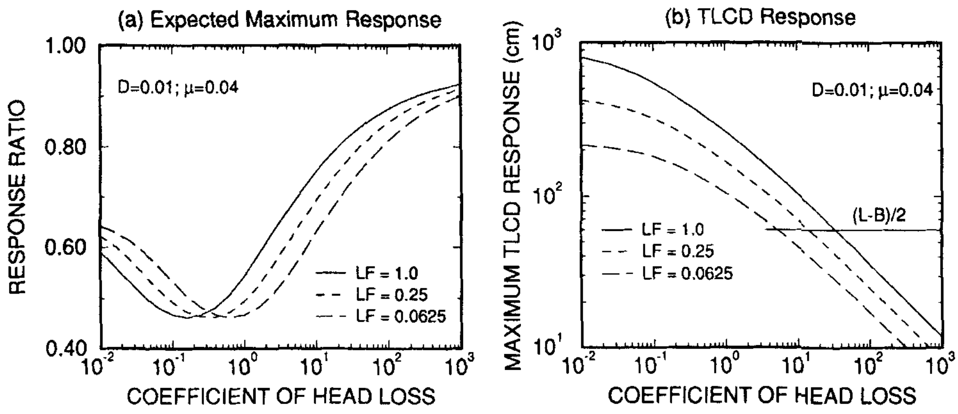


Figure 7. (a) Stuctural response reduction ratios and (b) expected maximum TLCD displacements for various loading intensities

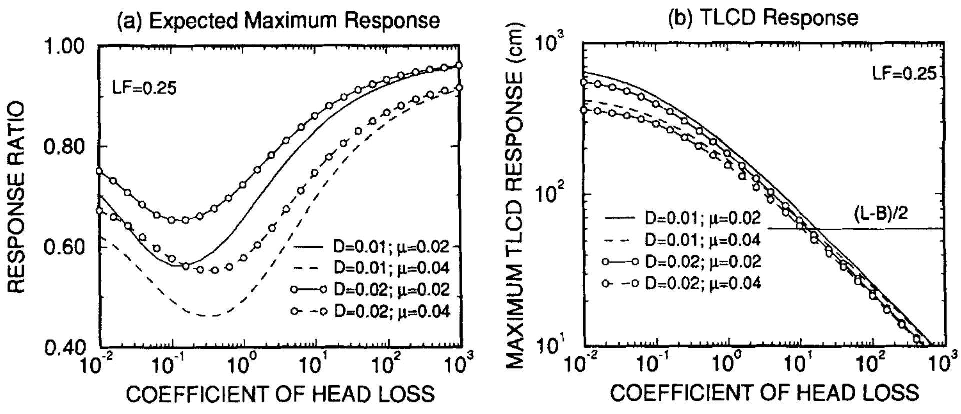


Figure 8. (a) Structural response reduction ratios and (b) expected maximum TLCD displacements for two mass ratios and structural damping ratios

cent structural damping ratios, and a loading intensity factor of 0·25. In general, the optimum coefficient of head loss increases as the mass ratio increases but is relatively insensitive to the structural damping ratio for lightly damped structures. As before, optimal TLCD performance cannot be achieved because of the limitation on the maximum displacements of the liquid column.

The response of an MDOF shear-beam type structure that represents a 30-storey building with the modal properties shown in Table IV is investigated for the earthquake loading under consideration with a load factor LF $= 0{\cdot}444$. For this input ground motion the expected maximum ground acceleration is two-thirds of the peak ground acceleration in the S60E-SCT record of the 1985 Mexico City earthquake. Since the modal frequencies are well separated, this structure responds primarily in the first mode under this earthquake loading. Time-histories of the RMS modal displacement for the first mode are shown in Figure 9(a) for the structure without control and with a TLCD placed at the top storey. The damper properties are: $B/L = 0{\cdot}7$, $\gamma = 1{\cdot}0$ and $\mu = 0{\cdot}02$, together with the coefficients of head loss indicated in Figure 9. The RMS displacements of the liquid column are shown in Figure 9(b). Since the response of the TLCD is non-linear, a validation of the equivalent linearization technique with Monte Carlo simulation is appropriate and is done for this example. The RMS displacement response for the first mode and for the liquid column damper that are obtained with the equivalent linearization are compared with Monte Carlo simulation results (20 samples) in Figures 9(c) and 9(d) for two values of the coefficient of head loss, ξ. A good agreement is observed between the equivalent linearization and the simulation results.

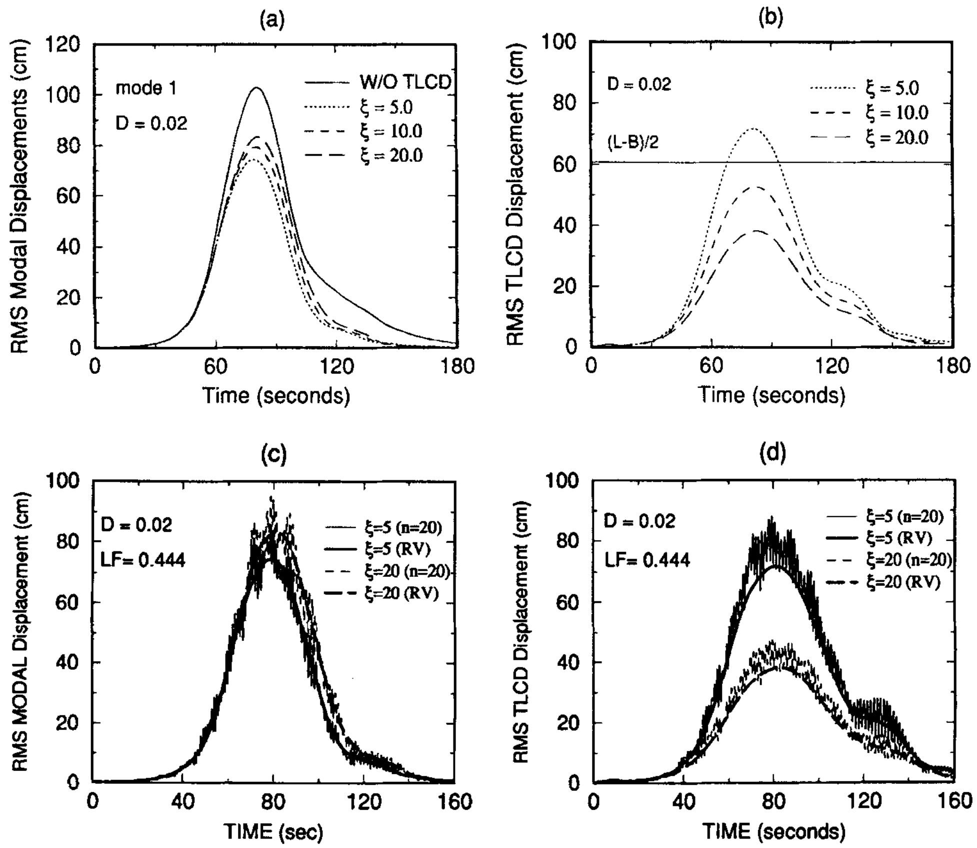


Figure 9. MDOF shear-beam building: (a) RMS modal displacements (mode 1); (b) RMS TLCD displacements; (c) RMS modal displacements from random vibration analysis (RV) and simulation ($n = 20$); and (d) RMS TLCD displacements from random vibration analysis (RV) and simulation ($n = 20$)

The maximum value of the RMS interstorey shears and absolute accelerations of the various storeys are shown in Figures 10(a) and 10(b), respectively. Even though the structural damping ratio is equal to 2 per cent, and the coefficient of head loss and tuning ratio are not optimal, a significant reduction in structural response can still be predicted.

The time-history of the RMS modal displacement response for the second and higher modes, not shown here, indicate that the presence of the TLCD may increase the modal displacement responses for the second and higher modes. However, since those modes do not contribute significantly to the total displacement response this spillover effect does not have an adverse effect on the TLCD performance for the example under consideration.

Short-duration loading

A stochastic input ground motion with a strong motion duration much shorter than that considered above is used. The integrals of the squared accelerations for the two ground motions considered, which are shown in Figure 4, clearly indicate the more marked transient nature of the 1968 Hachinohe record. The TLCD performance expressed in terms of the expected maximum structural displacement reduction ratio is illustrated with an SDOF structure with a natural frequency, $\omega_s = 2{\cdot}4$ rad/sec and a structural damping ratio of 1 per cent, for a TLCD with $B/L = 0{\cdot}7, \gamma = 1{\cdot}0$ and $\mu = 0{\cdot}04$. The structural response ratios achieved for three loading intensities are shown in Figure 11(a). Although the earthquake ground motion is of

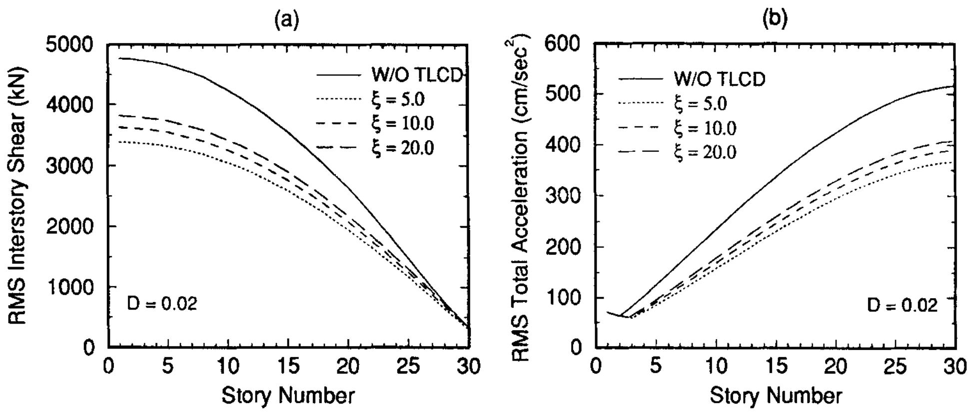


Figure 10. MDOF shear-beam building: (a) maximum RMS interstorey shears; (b) maximum RMS total accelerations

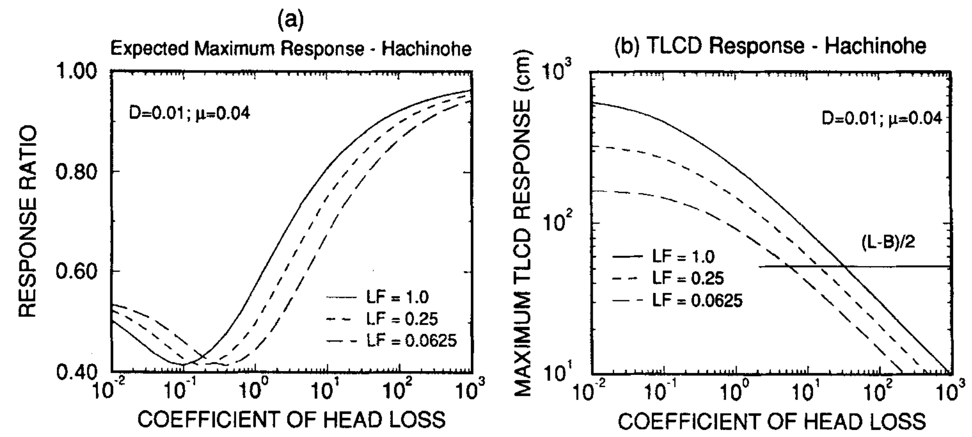


Figure 11. (a) Structural response reduction ratios and (b) expected maximum TLCD displacements for various loading intensities (Hachinohe model)

considerable shorter duration than that considered above, still significant structural response reduction is predicted. As before, in order to maintain full inertial coupling optimum TLCD performance for this tuning ratio cannot be achieved.

The results shown in Figures 7 and 11 indicate that optimal coefficients of head loss are load-intensity dependent, namely, that for a given input ground motion the required values for the coefficient of head loss increase as the load intensity decreases. Since the coefficient of head loss depends on the orifice opening ratio, and the earthquake loading intensity is not known *a priori*, this seems to indicate that active control of the TLCD orifice opening may lead to a better TLCD performance.

CONCLUSIONS

The seismic performance of TLCDs is evaluated using time-domain random vibration analysis. A parametric study is conducted to evaluate the sensitivity of their performance with respect to various design parameters, namely, the mass ratio, coefficient of head loss and tuning ratio, structural damping, and the loading intensity and duration. It is observed that there exist optimal values of the coefficient of head loss and tuning ratio for which optimal TLCD performance may be achieved for a given mass ratio. Such optimal TLCD parameters will also depend on the intensity, duration and frequency content of the loading. The random vibration analysis can be used to compute the mean-square response statistics and also their derivatives with respect to the critical design parameters. Those analytical tools may then be used in conjunction with optimization algorithms to determine the optimal TLCD design parameters for a given structure and loading condition. It is shown that optimal TLCD performance may not be achieved if the required TLCD length is not sufficient to accommodate the expected maximum TLCD displacements without loss of full inertial coupling. However, even without optimum design, TLCDs appear to be promising passive control devices for structural periods longer than about 2·5 sec and for ground motions similar to the stochastic ground motion processes considered. Additional studies are underway to investigate the efficiency of active TLCD control and for ground motions with different characteristics.

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